Lower Secondary School Mathematics Teachers' Topic-Specific Content Knowledge in the U.S. and Russia

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Abstract
This interpretive cross-case study examined the U.S. and Russian teachers' topic-specific knowledge of lower secondary mathematics. In total, N=16 teachers (8 from the U.S., and 8 from Russia) were selected for the study using non-probability purposive sampling technique. Teachers completed the Teacher Content Knowledge Survey (TCKS) as part of the purposive selection. The survey consisted of multiple-choice items measuring teachers' content knowledge at the cognitive levels of knowing, applying, and reasoning. Teachers were also interviewed on the topic of fraction division using questions addressing their content and pedagogical content knowledge. In order to analyze the qualitative data, we conducted meaning coding and linguistic analysis of teacher narratives as primary methods of analysis.
The study revealed that there are explicit similarities and differences in teachers' content knowledge as well as its cognitive types. The results are reflected in meanings expressed and language used by teachers while responding to topic-specific questions on the division of fractions. The results of the study suggest that in the cross-national context teachers' knowledge could vary depending on curricular as well as socio-cultural priorities placed on teaching and learning of mathematics.
The study's main findings contribute to the body of literature in the field of cross-national research on teacher knowledge with a narrow focus on a topic-specific knowledge. It suggests close comparison and learning about issues related to teacher knowledge in the U.S. and Russia with a potential focus on re-examining practices in teacher preparation and professional development.
Keywords: topic-specific content knowledge, teacher knowledge, lower secondary school mathematics, cross-national comparison.

Предметно-тематические знания учителей математики средней школы в США и России

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Аннотация

Данное исследование проводилось в США и в России и сравнивало знания учителей по математике на уровне средней школы. Всего восемь учителей из США и восемь учителей из России были выбраны с помощью детерминированной выборки. Педагоги заполняли опрос, состоящий из вопросов с несколькими вариантами ответа. Этот опрос был создан для выявления объема знаний преподавателей с использованием категории когнитивных процессов (знание, применение и рассуждение). Дополнительно с учителями проводилось интервью на тему деления дробей, чтобы определить их понимание и объем знаний. Для анализа качественных данных мы кодировали основные идеи, а также проводили лингвистический анализ высказываний педагогов.

Анализ данных показал, что существует четкая зависимость между объемом знаний и когнитивными способностями учителей из этих стран. Результаты основаны на суждениях педагогов и на формулировках, использованных при ответах на вопросы про деление дробей. В итоге можно предположить, что знания учителей зависят от учебного плана и социально-культурных особенностей изучения и преподавания математики.

Результаты исследования вносят вклад главным образом в развитие межнациональных исследований и в меньшей степени в область математических знаний педагогов. На примерах России и США осуществляется детальное осмысление проблем, связанных со знаниями учителя, потенциально акцентируется внимание на пересмотре методик профессиональной и педагогической подготовки учителя.

Ключевые слова: предметно-тематическое знание, педагогическое знание, математика в младшей и средней школе, межнациональное сравнение.

Introduction

Cross-national studies allow understanding of how teacher education is contextualized in selected countries which requires “a range of analytical methods that draw out conflicting views, contested areas and shared beliefs” (LeTendre, 2002). In the last decade, a number of cross-national studies on teacher education focused on unpacking “culturally contextualized and semantically decontextualized dimensions” in order to create “a more balanced comparative perspective” in teacher preparation across countries (Kim Ewha, Ham, Paine, 2011). Scholars have addressed characteristics such as teachers’ perceptions of effective mathematics teaching (Cai, Wang, 2010), the role of opportunity to learn in teacher preparation (Schmidt, Cogan, Houang, 2011), teacher education effectiveness (Blomeke, Suhl, Kaiser, 2011), teachers’ epistemological beliefs on nature of mathematics (Felbrich, Kaiser, Schmotz, 2012), and other issues. A number of papers addressed these issues at the pre-service teacher preparation level (Tatto, Senk, 2011; Felbrich, Kaiser, Schmotz, 2012). However, few comparative studies focused on in-service teachers’ content knowledge. Moreover, the field lacks research that provides an in-depth analysis of teacher knowledge at a topic-specific level. This study attempts to fill the gap and aims to examine the U.S. and Russian in-service teachers' content knowledge through the lens of topic-specific context – division of fractions. Considering the importance of teachers' topic-specific knowledge, the study focused on the following research question: to what extent is the U.S. and Russian lower secondary mathematics teachers' content knowledge similar and/or different in the topic-specific context?

The paper is several sections. First, we provide extended literature review in the field of cross-national studies in teacher education and teacher knowledge. Then we discuss the methodology of the study which consists of the research design, participants, procedure, data collection and analysis. Finally, we will present the results of the study followed by discussion and conclusion.

Cross-national Studies in Teacher Education and Teacher Knowledge

In this literature review, we will discuss recent studies in teacher education within the cross-national context. Studies vary in scope addressing different issues including but are not limited to general aspects in teacher education, teacher knowledge, different types of teacher knowledge, connections between teacher knowledge and student performance, instrument development and adaptation, to name a few.

Within the general aspects of teacher education, scholars emphasized "how the profession has been impacted by the forces of globalization. The impact varies in different contexts, according to local factors" (Adamson, 2012). Studies suggest that international comparisons in teacher education should be sensitive to the social, historical, and cultural contexts. As an example of this approach, one can consider the study that examined educational aims and curricula of several mathematics teacher preparation programs in South Korea and the U.S. (Ewha, Ham, Paine, 2011). The study concluded that "transnational commonalities and national differences exist simultaneously in social expectations for teacher knowledge" (Ewha, Ham, Paine, 2011).

Additionally, studies addressed significant cultural differences in the effectiveness of teacher education programs across the globe. Blomeke, Suhl, Kaiser (2011) examined the effectiveness of teacher education programs in 15 countries by analyzing the following indicators: teachers' test scores and its variability across different characteristics such as gender and language which could impact "differential choices of teacher education programs according to background and differential achievement of teachers from these programs" (Blomeke, Suhl, Kaiser, 2011). It was reported that teacher education programs in different countries offer opportunities to learn consistent with their vision of what teachers need to know and be able to do in mathematics classroom. In this regard, the study by Blomeke (2012) reveals how diverse yet homogeneous teacher education programs across countries could be.

Another important topic in the literature on cross-national studies in teacher education is an issue of teacher content knowledge and pedagogical content knowledge. Kleickmann, Richter, Kunter, Elsner, Besser, Krauss, Baumert (2013) discuss the role of structural differences in teacher education programs in Germany with regard to the development of teacher content and pedagogical content knowledge. They claim that pedagogical content knowledge as well as content knowledge are critical components of teacher competence that impact student performance. At the same time, little is reported about how structure of teacher education program may affect the development of teacher content knowledge and pedagogical content knowledge. The key finding of this study (Kleickmann, Richter, Kunter, Elsner, Besser, Krauss, & Baumert, 2013) shows that significant difference in teacher content and pedagogical content knowledge was reported between pre-service teachers at the beginning and the end of initial teacher education program. The study by An, Kulm, and Wu (2004) compared the pedagogical content knowledge of middle school mathematics teachers in the U.S. and China and found that it differs markedly, which consequently has an impact on teaching practice. While Chinese teachers emphasized development of procedural and conceptual knowledge through rigid practices, the U.S. teachers focused on a selection and implementation of a variety of activities to foster inquiry in mathematics classroom.

Few studies aimed at an international comparison of the effects teacher mathematics knowledge and pedagogy have on student achievement (Baumert, Kunter, Blum, Brunner, Voss, Jordan, Klusmann, Krauss, Neubrand, & Tsai, 2010; Marshall & Sorto, 2012). Baumert et al. (2010) addressed concerns about professional knowledge teachers need to deliver high-quality instruction. The study conducted in Germany emphasized the significance of teacher content knowledge and pedagogical content knowledge not only for high-quality instruction but also for student success in secondary school mathematics. The study key results indicated significant effect of teachers' pedagogical content knowledge on students' achievement through cognitive activation and individual support in learning mathematics. In the study that took place in Guatemala, Marshall and Sorto (2012) raised an important question: "why are some teachers more effective than others?" In order to answer this question, Marshal and Soto (2012) emphasized understanding of the interplay between teacher preparation, teacher knowledge and pedagogy, and student performance. The main results of the study suggested that effective teachers have different, more effective, kinds of mathematical knowledge that impacts student learning and understanding.

It is worth mentioning in the literature review section that a good number of cross-national studies have been conducted at the level of future pre-service teachers due to the large scale Teacher Education and Development Study in Mathematics (TEDS-M) that included several countries around the world (Senk, Tatto, Reckase, Rowley, Peck, & Bankov, 2012; Blomeke, Paine, Houang, Hsieh, Schmidt, Tatto, Bankov, Cedilllo, Cogan, Han, Santillan, & Schwille, 2008; Tatto, Senk, 2011). Senk et al. (2012) reported the results of the comparative study on future teachers' mathematical knowledge for teaching which was measured by the instrument (e.g. teacher knowledge survey) that was developed, translated, adapted, and validated in field trials in participating countries. The study reported differences in the structure of teacher preparation programs as well as differences in teachers' content knowledge and pedagogical content knowledge within and between teacher education programs and countries. The study by Blomeke and colleagues (2008) focused on cross-national comparison of teachers' general pedagogical knowledge in Germany, South Korea, Taiwan, and the U.S. with the explicit goal of assessing teachers' competence to plan a lesson. Since the participating countries have distinctively different teacher-education systems, it was reflected in reported significant differences in future teachers' general pedagogical knowledge at the beginning and at the end of teacher education programs. The study suggested thick cultural discourses in interpreting differences between teacher education programs in participating countries.

Additionally, a body of literature on cross-national research includes studies on instrument development and adaptation (Andrews, 2009; Delaney, Ball, Hill, Schilling, & Zopf, 2008; Tatto, & Senk, 2011). Delaney et al. (2008) reported results of the study on adaptation of the U.S. based measure - Mathematical Knowledge for Teaching – for the use in Ireland. It is valid to make an assumption that the mathematical knowledge used by teachers in different countries is not the same. Nonetheless, the study reported overlap between the knowledge that is used to teach in both countries. At the same time, the study suggested "the usefulness of conducting extensive checks on the validity of items used in cross-national contexts" (Delaney et al., 2008). In study conducted by Tatto and Senk (2011), authors present methodology from the TEDS-M study that included 17 participating countries. In this study, the instrument was developed to measure future teachers' mathematical content knowledge based on different content domains (e.g. number, algebra, geometry, data and chance) as well as cognitive domains (e.g. knowing, applying, and reasoning). Andrews (2009) conducted comparative study on mathematics teachers' observable learning objectives using videos of classroom teaching. His analysis revealed that despite the fact that videos have become an increasingly used tool in comparative research, methods of its analysis and coding are not developed well. The study included classroom observations and videos of mathematics classrooms in five European countries. The study results confirmed that even though the descriptors for classroom observations were consistently operationalized, analysis reported differences in teachers' conceptualization of mathematics teaching which brings importance of discussion on "the national mathematics teaching script" (Andrews, 2009).

A critically important domain in cross-national studies are teacher conceptions and beliefs. Cai and Wang (2010) researched the U.S. and Chinese teachers' cultural beliefs and conceptions about effective mathematics teaching. The scholars found that while teachers share common beliefs, they think differently about important characteristics of effective teaching. Whereas the U.S. teachers emphasized learning with concrete examples, the Chinese teachers capitalized on abstract reasoning after presenting concrete examples. Another insightful difference was reported on the affective domain of teaching: while the U.S. teachers value student participation, classroom management and a sense of humor, the Chinese teachers believe in value of content knowledge and study of textbooks. As in some previous studies, Cai and Wang (2010) suggested that differences and similarities in teachers' beliefs and conceptions should be discussed in a cultural context. The same approach is shared by Felbrich, Kaiser, and Schmotz (2012) in the cross-national study on cultural dimension of beliefs with the goal of examining pre-service elementary teachers' epistemological disposition toward the nature of mathematics. It is well documented that beliefs are critical part of teachers' professional competencies. The reported data were aiming to explore to what extent teacher beliefs are influenced by cultural factors such as individualism/collectivism.

Several cross-national studies targeted teaching practice and mathematics learning where scholars shared mathematics lessons from different countries. For instance, Seaberg (2015) conducted international comparison in Sweden and Finland and highlighted differences and similarities in practice of mathematics teaching and learning among these two countries and the U.S. In the review conducted by Ng and Rao (2010) the structure of Chinese number words is explored within linguistic and cultural context of mathematics learning. Particularly, scholars evaluated the role of language in students' mathematical understanding of number words and its potential impact on students' performance in cross-national studies of mathematics achievement. Authors claimed that number words in the Chinese language afford benefits for students' mathematics learning and understanding while highlighting interrelationships among language, culture, and mathematics learning. Schmidt, Cogan, Houang (2011) examined the role of the construct - opportunity to learn - in teacher preparation programs in an international context. Authors raised an important question – "given the finite time available, what sort of balance is provided for course work across the areas of mathematics content, mathematics pedagogy, and general pedagogy?" (Schmidt, Cogan, & Houang, 2011). Results of the study showed differences in teacher preparation programs in participating countries across the three areas. The study findings provided evidence to support policy decisions on evaluating quality of teacher preparation programs in an international context.

Analysis of a body of literature in cross-national research in teacher education and teacher knowledge demonstrates that few comparative studies focused on in-service teachers' content knowledge. More specifically, the field of cross-national research lacks studies that provide a close evaluation of teacher knowledge at a topic-specific level. Addressing this deficiency, the proposed study attempts to qualitatively examine the U.S. and Russian lower secondary school mathematics teachers' content knowledge in the topic-specific context – the division of fractions.

Methodology

In this section, we present the research design followed by the discussion on the selection of participants, data collection and analysis as well as instrument adaptation.

Research Design

We selected the interpretive cross-case study design to examine the U.S. and Russian teachers' topic-specific knowledge of one of the important themes in lower secondary mathematics curriculum in both countries - division of fractions. Merriam (1998) classified case studies with regard to its' overall intent as descriptive, interpretive, and evaluative. According to Merriam (1998), a descriptive case study presents "a detailed account of the phenomenon under study" (p. 38), an evaluative case study aims at "description, explanation, and judgement" (p. 39), and, finally, an interpretive case study focuses on "analyzing, interpreting, or theorizing about the phenomenon" (p. 38). Following on the interpretive case study design, 16 teachers (eight from each country) were selected for the study after completion of the Teacher Content Knowledge Survey (TCKS).

Instrument Translation and Adaptation

Initially, the TCKS instrument was developed, field tested, and validated in the USA (Tchoshanov, 2011). The survey consisted of 33 multiple-choice items addressing main topics of lower secondary mathematics curriculum: Arithmetic (9 items), Algebra (9 items), Probability and Statistics (6 items), Geometry and Measurement (9 items), as well as different cognitive types of content knowledge: facts and procedures (10 items), knowledge of concepts and connections (13 items), and knowledge of models and generalizations (10 items). Specification table along with item analysis was performed to ensure content and construct validity of the TCKS along with its' reliability measured by the Cronbach alpha coefficient at 0.839 (Tchoshanov, 2011).

Considering that teaching is a cultural activity (Stigler & Hiebert, 1998), one should be sensitive to issues related to adaptation of an instrument in different settings. Scholars (Andrews, 2011; Pepin, 2011) documented variations across countries in various ways curriculum and content are structured, procedures and concepts are introduced, assignments of homework as well as individual and group work in the classroom are distributed, blackboard is used during instruction, etc. Scholars apply different methods to validate and adapt an instrument in new setting. Delaney et al. (2012) employed the method with the following components: teacher interviews to explore the consistency of teacher thinking and answer choices made; factor analysis of the teacher responses to evaluate the structure of factors supporting the instrument domains; and analysis of video recordings of lessons to examine the relationship between the teachers' scores and teaching practice. Moreover, validity of an instrument heavily depends on the translation quality and linguistic equivalence (Pena, 2007). Therefore, we employed multi-level translation procedure using an expertise of the Russian- speaking members of the research team to ensure linguistic equivalence of the adapted TCKS items with two rounds of independent translations followed by the round of reconciliation.

Aside from taking TCKS, selected teachers were also interviewed on the topic of fraction division using questions addressing their content and pedagogical content knowledge. The cross-case analysis of teachers' topic-specific knowledge was conducted using meaning coding and linguistic analysis techniques (Kvale & Brinkmann, 2009).

Participants

A non-probability purposive sampling technique was employed to select study participants. Purposive sampling required that selected the U.S. and Russian teachers represent different quartiles of the total scores on the TCKS measure. It was also required that selected teachers teach at similar school settings (e.g. urban public schools).

With regard to the first criterion, the TCKS was administered to the initial sample of lower secondary mathematics teachers in USA (grades 6-9, N=102) and Russia (grades 5-9, N=97) (Tchoshanov, 2011; Tchoshanov, Cruz Quinones, K. Shakirova, Ibragimova & L. Shakirova, 2017). The initial sample from both countries was subdivided by quartiles using teachers' overall TCKS scores. The distribution of the U.S. and Russian teachers' TCKS scores by quartiles is presented in table 1.

Table 1 indicates that distribution of teachers across quartiles was similar with a third of the teachers in both the U.S. and Russian samples located in quartiles 1 and 3. There were 22% of the U.S. and 23% of Russian teachers located in the quartile 2 and 19% of the teachers in each country located in the quartile 4. We selected two teachers from each quartile after applying the purposive sampling criteria. Hence, the total study sample consisted of N=16 teachers (eight teachers from each country) who met the requirements of the purposive sampling. Selected teachers pseudonyms along with their total scores on TCKS across corresponding quartiles are presented in table 2.

Table 1. *Distribution of the U.S. and Russian teachers' total TCKS scores by quartiles*

Quartile	US teachers (N=102)			Russian teachers (N=97)		
	Range	N	%	Range	N	%
Q1	4-15	30	29	13-18	28	29
Q2	16-19	22	22	19-20	23	23
Q3	20-24	31	30	21-23	28	29
Q4	25-30	19	19	24-27	18	19

Table 2. *Selected USA and Russian teachers' total TCKS scores by quartiles*

Quartile	US Teachers		Russian Teachers	
	Pseudonym	Score	Pseudonym	Score
Q1	Rich Mary	13 15	Lera Inna	13 16
Q2	Grace Mark	18 19	Zina Victor	18 20
Q3	Lori Kate	21 23	Kiril Gala	21 22
Q4	Ron Sara	26 28	Anna Igor	25 27

Both the U.S. and Russian participants have similar teaching assignments – lower secondary school mathematics with content addressing the following main objectives: Arithmetic, Algebra, Probability and Statistics, Geometry and Measurement. All selected teachers teach in urban public schools.

Data Collection

The study used the following data source - structured teacher interviews on the topic of division of fractions. Teachers were interviewed using the following five questions related to the topic:

When you teach fraction division, what are important terms, facts, procedures, concepts, and reasoning strategies your students should learn?

What is the fraction division rule?

Apply the rule to the following fraction division problem: $1\frac{3}{4} \div \frac{1}{2} =$

4) Construct a word problem for the given fraction division: $1\frac{3}{4} \div \frac{1}{2} =$.

5) Is the following statement $\frac{a}{b} \div \frac{c}{d} = \frac{ac}{bd}$ (*a, b, c,* and *d* are positive integers) ever true?

The first question aimed at teachers' pedagogical content knowledge and focused on teachers' understanding of learning objectives for the topic of fraction division. The subset of questions (2-5) assessed teachers' understanding of topic-specific content across the cognitive domain.

Data Analysis

Most of the large-scale cross-national studies on student achievement (e.g. TIMSS, PISA) as well as teacher preparation (e.g. TEDS-M) focused on complex data collection and employ, primarily, quantitative methods for data analysis. However, "to fully understand how achievement is contextualized in a given nation requires not only sets of complex data but also a range of analytical methods that draw out conflicting views, contested areas and shared beliefs" (LeTendre, 2002). The proposed study consisted of two stages: 1) quantitative stage was used for the purpose of selection of teacher sample; 2) qualitative stage was applied to analyze teacher responses on a set of open-ended questions on the division of fractions. For the first stage, quantitative data from previous studies (Tchoshanov, 2011, 2017a, 2017b, 2017c) was used to further zoom into qualitative analysis of unpacking shared approaches as well as to address contested areas in teachers' topic-specific content knowledge in the U.S. and Russia.

Taking into account categorical nature of the quantitative data (e.g., frequency counts) collected in the study, we used a frequency comparison technique to compare responses of two independent groups of teachers to questions on the division of fractions.

During the qualitative stage, teacher interviews were audio recorded and transcribed. In order to analyze qualitative data, we conducted meaning coding and linguistic analysis of teacher narratives as a primary method of analysis (Kvale & Brinkmann, 2009). The linguistic analysis technique unpacks "the characteristic uses of language, … the use of grammar and linguistic forms" (Kvale & Brinkmann, 2009, p. 219) by participating teachers within the specific topic of lower secondary mathematics. Additionally, the linguistic analysis was applied to check teacher use of mathematical terminology (questions 1-3). In order to "breaking down, examining, comparing, conceptualizing and categorizing data" (Strauss & Corbin, 1990, p. 61) we used data-driven meaning coding technique. This technique was applied to analyze teachers' responses on questions tapping into their understanding of meanings of the division of fractions (question 4) as well as their justification for solving the non-routine problem (question 5). To increase the credibility of the qualitative data analysis, the meaning coding and linguistic analysis were performed and cross-checked by two independent raters.

Results

In this section, we present the U.S. and Russian teachers' responses to the questions on division of fractions.

Findings of the Quantitative Stage

Teacher Responses to Question 1

The question 1 asked "When you teach fraction division, what are important terms, facts, procedures, concepts and reasoning strategies your students should learn?" Accordingly, teacher responses were coded using the following categories: 1) vocabulary, 2) facts and procedures, 3) concepts and connections, and 4) reasoning. Frequency of teacher responses in each category are presented in table 3.

Table 3. *Frequency of the U.S. and Russian teachers' responses to Question 1 by categories*

Category	US teachers	Russian teachers
Vocabulary	33	38
Facts and Procedures	27	32
Concepts and Connections	20	23
Reasoning	0	6

The most frequently used category in response to question 1 was "vocabulary" with the total amount of counts = 71: 33 counts in the U.S. teachers' responses and 38 counts in Russian teachers' responses with no significance observed between the groups (chi-square $\chi^2=2.003$). Most frequently used terms emerged from teachers' responses are "division" (9 counts), "reciprocal" (11 counts), "denominator" (8 counts), "multiplication" (7 counts). Least frequently used terms are "dividend" (3 counts), divisor (3 counts), "quotient" (3 counts). With regard to categories "facts and procedures" and "concepts and connections", we did not detect any significant differences between the groups. The only category where an important difference was detected is the category of "reasoning". In the Findings of the Qualitative Stage section of the paper we will discuss these findings in more detail.

Teacher Responses to Question 2

The second question asked teachers to respond to the following: what is the fraction division rule? In table 4 we present frequency of terms used by the U.S. and Russian teachers while explaining the rule for fraction division along with chi-square values for each reported term.

Table 4. *Frequency of terms used by the U.S. and Russian teachers in response to Question 2*

Terms used by teachers	US teachers	Russian teachers
Flip	7	1
Reciprocal	7	8
Dividend	0	6
Divisor	0	6
First fraction	6	2
Second fraction	6	2
Quotient	0	1

All U.S. and Russian teachers correctly responded to this question. However, the way they described the rule deserves a separate discussion which we will provide in the Conclusion section.

Teacher Responses to Question 3

As expected, teachers' responses to the procedural question 3 (divide two given fractions) were the least insightful. Most of the teachers in both groups silently performed the division on a scratch paper that was provided to every participant. All participating teachers correctly solved the given fraction division task. Slight differences were observed in the representation of the answer. Whereas all eight U.S. teachers wrote the answer in mixed number form as 3½, only two Russian teachers did the same. Five Russian teachers wrote the answer in decimal form 3.5 and one Russian teacher wrote the answer in both forms 3½–3.5. One observation deserves mentioning and further discussion later in the Conclusion section: one of the U.S. teachers illustrated the division by a pictorial model.

Teacher Responses to Question 4

The question 4 tapped into teachers' understanding of meaning(s) of the division of fractions while asking them to construct a word problem for the given fraction division problem. There are several distinct meanings of the division of fractions discussed by scholars. For instance, Fischbein et al. (1985) and Simon (1993) identified two main meanings for the division of fraction: quotitive (measurement) and partitive (part-to-whole). At the same time, Greer (1992) proposed to consider the "rectangular area" model within the partitive meaning of the fraction division. Later Ma (1999) included the rectangular model in a separate category, which she called "product and factors." Therefore, Ma (1999, p. 72) claimed that there are three main models and corresponding meanings to represent the division of fractions: measurement, partitive, and product and factors.

The question 4 was challenging to the U.S. teachers – only five teachers were able to construct a correct word problem compared to eight Russian teachers. An insightful observation was recorded in models used by teachers to construct a word problem which will be further discussed in the Conclusion section. In table 5, we include frequencies of meanings/ models used by the teachers to construct word problems.

Table 5. *Frequency of meanings of fraction division used by the U.S. and Russian teachers in response to Question 4*

Meanings of fraction division	US teachers	Russian teachers
Part-to-whole (partitive)	0	2
Measurement (quotitive)	5	2
Rectangular area model (product and factors)	0	4
Incorrect	3	0

The analysis showed a difference not only for the rectangular area model but also overall difference in performance of the U.S. and Russian teachers on this particular task.

Teacher Responses to Question 5

The question 5 aimed at assessing teachers' critical reasoning: is the following statement $\frac{a}{b} \div \frac{c}{d} = \frac{ac}{bd}$ (*a, b, c,* and *d* are positive integers) ever true? This question was challenging to both the U.S. and Russian teachers. Table 6 captures frequencies of solutions/ proofs proposed by teachers.

Table 6. *Frequency of solutions/ proofs proposed by the U.S. and Russian teachers in response to Question 4*

Teacher responses	US teachers	Russian teachers
Never true	5	4
True if a=b=c=d	1	1
True if c=d	1	3
No solution provided	1	0
Using numerical values to prove	4	0

As depicted in the table 6, we were not able to observe any significant differences between groups in a number of correct responses to question 5 (only one correct and one partially correct solution proposed from the U.S. teachers compared to three correct and one partially correct solutions provided by Russian teachers). However, an interesting observation was recorded with regard to a method of proof used by teachers which we will elaborate further on in the next section.

Findings of the Qualitative Stage

In this section, we discuss major qualitative results of the study. We will address some insightful observations related to every question we used during the Quantitative part of the study. We will start with observations on teacher articulation of the learning objectives for the topic of fraction division (question 1). Then we will discuss teacher use of mathematical vocabulary, facts and procedures (questions 1-3). We will proceed to teacher understanding of meaning(s) of the division of fractions. Finally, we will address the observation on methods employed by teachers while responding to question 5.

Teacher articulation of the learning objectives for the division of fractions

Most insightful finding in teachers' responses to question 1 was the fact that both U.S. and Russian teachers define learning objectives for the division of fractions in quite similar ways. Both groups clearly outlined main vocabulary students should learn, facts and procedures students should master, and concepts students should understand. The revealing difference was observed in teachers' response to the reasoning category. Despite the fact that the question 1 explicitly asked to articulate "what are important … reasoning strategies your students should learn?", none of the U.S. teachers responded to this part of the question compared to six Russian teachers who highlighted the importance of "developing logical reasoning" (4 teachers) as well as "checking for reasonableness" (2 teachers). This finding may suggest that the U.S. teachers do not see a "reasoning" potential in the topic of the division of fractions whereas their Russian counterparts emphasize the development of students' critical thinking as one of the important learning objectives for the topic of fraction division.

Teacher use of mathematical vocabulary related to the division of fractions

As mentioned earlier, both the U.S. and Russian teachers emphasized importance of developing students' mathematical vocabulary related to the topic of the division of fractions. Table 7 captures frequencies of terms used by teachers in both groups.

Table 7. *Frequency of vocabulary terms used by the U.S. and Russian teachers in response to Question 1*

	Vocabulary terms	US teachers	Russian teachers
1.	Parts of a whole	3	3
2.	Division	6	3
3.	Numerator	1	5
4.	Denominator	3	5
5.	Reciprocal	4	7
6.	Improper fraction	1	4
7.	Mixed number	1	4
8.	Multiplication	3	4
9.	Multiplicative inverse	2	6
10.	Factor/ Product	0	3
11.	Dividend	0	3
12.	Divisor	0	3
13.	Quotient	0	3

Between two groups of teachers, there were 13 terms recorded in response to the "vocabulary" category of the question 1 as indicated in table 7. We thought that several observations deserve a further discussion. First, most frequently used term among the U.S. teachers was "division" (6 frequency counts) whereas "reciprocal" (7 counts) and "multiplicative inverse" (6 counts) were the most frequently used terms by Russian teachers. This result may suggest that the U.S. teachers focused on the operation in general (e.g. division) whereas Russian teachers emphasized the operation specific to the division of fractions (e.g. reciprocal, multiplicative inverse). We also noticed that Russian teachers were using the terms reciprocal and multiplicative inverse interchangeably. It may suggest that Russian teachers use these synonyms with some level of distinction. Indeed, from mathematical perspective, the term «reciprocal» has a specific meaning: the reciprocal of x is $1/x$. For instance, the reciprocal of 2 is 1/2 the same way as the reciprocal of 1/2 is 2. At the same time, the term «multiplicative inverse» is more general for the very reason that the term "inverse" means something that is opposite to something. For example, subtraction is an inverse operation to addition the same way as multiplication is an inverse operation to division. Therefore, we distinguish between terms additive inverse and multiplicative inverse.

The second observation concerns the elements of the division operation. Even though the term division as an operation was most frequently used by the U.S. teachers, none of them reported elements of this operation in their responses. In contrast, three Russian teachers referred to the elements of the division operation (e.g. dividend, divisor, and quotient) as an important learning objective to reinforce while studying the division of fractions.

Accurate use of mathematical terms by Russian teachers was also evident in the response to question 2. Even though all U.S. and Russian teachers correctly responded to this question, the way they described the rule deserved a close examination. First, we were pleased to observe that despite of low frequency in using terms "reciprocal" and "multiplicative inverse" in response to question 1, the U.S. teachers recalled the term "reciprocal" more frequently (7 counts) in response to question 2. Next observation is concerned with the use of accurate mathematical terminology: "dividend" vs. "first fraction" and "divisor" vs. "second fraction" which was statistically significant in both cases as depicted in table 4. Third, our observation revealed a strong tendency on the part of the U.S. teachers to use the term "flip" as a sub-language for reciprocal/ multiplicative inverse. Last but not least, we were pleased to report the pictorial representation of fraction division performed by Kate - the U.S. teacher - in response to question 3 using measurement model of fraction division.

Teacher understanding of meaning(s) of the division of fractions

Following on the previous discussion on Kate's visual representation of the measurement model of fraction division, we found that the measurement model was the most popular model (5 frequency counts as presented in table 5) and the only one model used by the U.S. teachers in response to question 4 asking to construct a word problem for the given problem $1\frac{3}{4} \div \frac{1}{2} =$. In contrast, Russian teachers applied all three models for the fraction division meaning proposed by Ma (1999) with the product and factors/ rectangular area model being the significant one.

Teacher reasoning in the fraction division context

Analysis of teacher narratives to question 5 did not show significant differences between groups in number of correct responses. Whereas the U.S. teachers proposed only one correct (c=d) and one partially correct solution ($a=b=c=d$), their Russian counterparts provided three correct and one partially correct solutions. A significant difference was reported with regard to a method of proof used by teachers. None of the Russian teachers attempted to proof the statement numerically compared to four U.S. teachers who tried to plug in different numbers to check if the statement works. Also, there was one episode of not offering any solution to question 5 among the U.S. teachers which was not a case among Russian teachers.

Conclusion

Synthesizing main findings of the study, we report that topic-specific level of analysis helped us to unpack hidden insights in terms of differences and similarities in teacher knowledge among participants in the U.S. and Russia. Considering qualitative nature of the research design, we are cognizant of limitations of the study and, congruently, we are sensitive to not overgeneralize its results. Granualized methodology used in the study to unpack and analyze teacher topic-specific knowledge could be considered as a potential contribution to the field of cross-national studies on teacher knowledge.

Overall, the study findings revealed that there are similarities and differences in teachers' content knowledge as well as its cognitive types. The results are reflected in meanings expressed and language used by teachers while responding to topic-specific questions on the division of fractions. The results of the study suggest that in the cross-national context teachers' knowledge could vary depending on curricular as well as socio-cultural priorities placed on teaching and learning of mathematics.

The study main findings contribute to a body of literature in the field of cross-national research on teacher knowledge with a narrow focus on a topic-specific knowledge. It suggests close comparison and learning about issues related to teacher knowledge in the U.S. and Russia with a potential focus on re-examining practices in teacher preparation and professional development. Findings of the study have implications to teaching practices and opportunities to learn at the level of lower secondary mathematics in the U.S. and Russia.

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